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Neutrino Cross Sections in a Quark Parton Model with the Weinberg Neutral Current

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ABSTRACT

A special version of the quark parton model involving the Cabbibo charged current and the Weinberg neutral current is used to calculate the charged-current neutrino and antineutrino total cross sections in the scaling region. The agreement with the present CERN data for both the cross section ratio and slopes is excellent. Predictions are given for the neutral-current induced cross sections.

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The possible role of weakly-coupled neutral currents in the strangeness-conserving neutrino reactions remains unclear. Theoretical lower bounds for neutrino cross sections involving neutral currents of the Weinberg-type[1] have been placed by several groups of authors.[2] On the experimental side, some inconclusive evidence for neutrino reactions of the neutral-current variety has accumulated,[3] but the problem of background from neutron-induced reactions simulating neutral-current events remains a serious one. It would be of considerable help to the experimentalists if one could predict theoretically the expected anti-neutrino-to-neutrino cross section ratio for the neutral current events. We attempt to do that here by considering the Weinberg neutral current in the framework of the quark parton model.[4]

In order to clarify our procedure and to distinguish it from the more general work of Budny and Scharbach,[5] and Riazuddin and Fayyazuddin[6] who have previously studied the Weinberg neutral current in the parton and light-cone frameworks, we cite below the main points. Details will be published elsewhere.[7]

1) We abstract the Weinberg neutral current from the 4-quark model[8] and write it in the form

$$J_\lambda^{(0)} = (V-A)_\lambda^{\ 3} - 2\sin^2\theta_W J_\lambda^{em}, \qquad (1)$$

where the first term is just the third component of the strangeness-conserving V - A current and θ_W is the Weinberg mixing angle which remains unspecified and determined only by experiment.[9]

2) We apply the Cabibbo charged current and the Weinberg neutral current to the triplet constituent quark model.[10] The transverse structure function of the electromagnetic current can be written as[11]

$$2F_T^e(x) = \sum_j D_j(x) Q_j^2, \tag{2a}$$

while the helicity structure functions of the charged current are given by

$$F_\pm^\nu(x) = \sum_j (1 \mp \epsilon_j)\, D_j(x) [\cos^2\theta_C\, I_j^2 + \sin^2\theta_C\, V_j^2]; \tag{2b}$$

for the helicity structure functions of the neutral current, we find

$$f_\pm^\nu(x) = \sum_j D_j(x) \{ (1 \mp \epsilon_j)\ [I_{3,j}^2 - 2\sin^2\theta_W\, I_{3,j}\, Q_j] + 2\sin^4\theta_W\, Q_j^2 \}, \tag{2c}$$

in terms of the quark distribution functions $D_j(x)$, $j = 1, 2, 3, -1, -2, -3$, and the signature factor ϵ_j which is +1 for quarks and -1 for antiquarks. The longitudinal structure functions $F_L(x)$, $F_o(x)$, and $f_o(x)$ receive contributions only from the gluons present in the nucleon.

3) We compute zeroth- and first-moment sum rules for the neutral-current structure functions $f_\pm(x)$ in terms of the average number of quarks of type j in the proton

$$\langle N_j \rangle = \int_0^1 dx\, D_j(x), \tag{3a}$$

and the average fractional longitudinal momentum carried by the jth quark in the proton,

$$d_j = \int_0^1 dx\, x\, D_j(x). \tag{3b}$$

Inequalities are obtained from these sum rules by requiring

$$\langle N_1 \rangle \geq 2, \qquad \langle N_2 \rangle \geq 1,$$
$$\langle N_j \rangle \geq 0, \qquad j = 3, -1, -2, -3$$

and

$$0 \leq d_j \leq \langle N_j \rangle$$

with

$$0 \leq \epsilon = 1 - \sum_j d_j < 1,$$

where ϵ is the average fractional longitudinal momentum carried by the gluons.

4) For simplicity, we require that the inequalities obtained be satisfied for all possible values of θ_W by the smallest average number of quarks carrying the largest possible average fractional momentum. This requirement leads to[7]

$$\begin{aligned} &\langle N_1\rangle = 2, && \langle N_2\rangle = 1, \\ &\langle N_{-1}\rangle = 0, && \langle N_{-2}\rangle = 0, \\ &\langle N_3\rangle = 3, && \langle N_{-3}\rangle = 3, \end{aligned} \tag{4a}$$

and

$$\begin{gathered} d_1 = d_2 = d_3 + d_{-3} = \tfrac{1}{3}(1 - \epsilon), \\ d_{-1} = d_{-2} = 0. \end{gathered} \tag{4b}$$

The picture of the proton which emerges from this analysis is that of three nonstrange valence quarks which distinguish proton from neutron together with a small sea of strange quark pairs. In particular, nonstrange antiquarks are absent. On the average there are just nine quark partons in the proton held together by gluons.[12] Comparison of (4a) and (4b) suggests that the strange quark pairs contribute more to the diffractive part of $D_j(x)$ on the average than do the nonstrange quarks. We shall not elaborate on this further, for this simple model has nothing to say about the wee partons.[4]

Instead, we focus on the first-moment results in (4b) which are of most immediate interest to us here. One can express the experimentally accessible quantities in terms of the d's according to

$$I^{ep} = \int_0^1 dx\ 2xF_T^{\ ep} = \frac{4}{9}(d_1+d_{-1}) + \frac{1}{9}(d_2+d_{-2}+d_3+d_{-3}), \qquad (5a)$$

$$I^{en} = \int_0^1 dx\ 2xF_T^{\ en} = \frac{4}{9}(d_2+d_{-2}) + \frac{1}{9}(d_1+d_{-1}+d_3+d_{-3}), \qquad (5b)$$

$$\begin{aligned}\sigma(\nu+N\to\mu^-+X) &= \frac{1}{2}(\sigma^{\nu p}+\sigma^{\nu n}) \\ &= \frac{G^2}{2\pi}s\left[\frac{1}{3}(d_{-1}+d_{-2}) + \cos^2\theta_C(d_1+d_2) + 2\sin^2\theta_C d_3\right], \qquad (5c)\end{aligned}$$

$$\begin{aligned}\sigma(\nu+N\to\nu+X) = \frac{G^2}{2\pi}s\Big\{&\frac{1}{2}(d_1+d_2) + \frac{1}{6}(d_{-1}+d_{-2}) \\ &- \left[d_1+d_2+\frac{1}{3}(d_{-1}+d_{-2})\right]\sin^2\theta_W + \frac{4}{27}\left[5(d_1+d_2+d_{-1}+d_{-2})\right. \\ &\left. + 2(d_3+d_{-3})\right]\sin^4\theta_W\Big\}; \qquad (5d)\end{aligned}$$

the antineutrino cross sections are obtained from the neutrino ones by replacing d_j by d_{-j}. In the above, G is the weak interaction constant, θ_C is the Cabibbo angle, and $s = 2ME$ in the lab system.

From Eqs. (4b), (5a), and (5b) we are led to the conclusion that

$$I^{ep} = I^{en} = \frac{2}{9}(1-\epsilon). \qquad (6)$$

The experimental data on inclusive electron scattering indicates, on the otherhand, that[13]

$$I^{ep}_{expt} = 0.18 \pm 0.018, \qquad I^{en}_{expt} = 0.12 \pm 0.012; \qquad (7)$$

i.e., within experimental accuracy the two moments are not equal in disagreement with (6). It is reasonable to assume, however, that the average value of $I^{ep} + I^{en}$ is predicted correctly but that the discrepancy arises from the departure of d_1 and d_2 from strict equality.[13] A value of

$$\epsilon \simeq 0.32 \qquad (8)$$

can then be deduced in good agreement with previous estimates for the gluon momentum contribution.[13]

If we allow a 20% departure from equality[14] for d_3 and d_{-3}, Eq. (5c) and its antineutrino counterpart together with (8) imply the results

$$\sigma_{ch}^{\nu N} \equiv \sigma(\nu+N\to\mu^-+X) = \frac{G^2}{2\pi}s\ (0.441 \pm 0.003), \tag{9a}$$

$$\sigma_{ch}^{\bar\nu N} \equiv \sigma(\bar\nu+N\to\mu^++X) = \frac{G^2}{2\pi}s\ (0.163 \pm 0.002), \tag{9b}$$

and

$$0.363 \le R^{ch} \le 0.377 \tag{9c}$$

for the ratio of the antineutrino to neutrino cross sections, $R = \sigma^{\bar\nu N}/\sigma^{\nu N}$. All are in excellent agreement with the present experimental results from CERN:[15]

$$\sigma_{expt}^{\nu N} = \frac{G^2}{2\pi}s\ (0.450 \pm 0.090), \tag{10a}$$

$$\sigma_{expt}^{\bar\nu N} = \frac{G^2}{2\pi}s\ (0.170 \pm 0.034), \tag{10b}$$

$$R_{expt}^{ch} = 0.377 \pm 0.023. \tag{10c}$$

Turning our attention to the neutral-current reactions, we obtain from Eqs. (4b), (5d), and the antineutrino counterpart of (5d)

$$\sigma_o^{\nu N} \equiv \sigma(\nu+N\to\nu+X) = \frac{G^2}{2\pi}s\left[\frac{1}{3} - \frac{2}{3}\sin^2\theta_W + \frac{16}{27}\sin^4\theta_W\right](1-\epsilon), \tag{11a}$$

$$\sigma_o^{\bar\nu N} \equiv \sigma(\bar\nu+N\to\bar\nu+X) = \frac{G^2}{2\pi}s\left[\frac{1}{9} - \frac{2}{9}\sin^2\theta_W + \frac{16}{27}\sin^4\theta_W\right](1-\epsilon). \tag{11b}$$

The present information from CERN[16] indicates that $\sin^2\theta_W \lesssim 0.60$ while the results of the Reines experiment[17] on $\bar\nu_e + e^- \to \bar\nu_e + e^-$ imply[18] $\sin^2\theta_W \lesssim 0.40$. The neutral-current cross sections, $\sigma_o^{\nu N}$ and $\sigma_o^{\bar\nu N}$, are quite sensitive to the value assumed for $\sin^2\theta_W$, so we have tabulated the results for values in the range $0 \leq \sin^2\theta_W \leq 0.60$ in Table I.

The cross section ratios are of special interest. The value of 0.27 for the ratio $\sigma_o^{\nu N}/\sigma_{ch}^{\nu N}$ is close to the lower limit of 0.23 estimated by Pais and Treiman[2] and Paschos and Wolfenstein[2] for $\sin^2\theta_W = 0.33$. This is the case because the isoscalar contribution which was neglected by those authors turns out to be small in the parton model considered here.

It is interesting to note that the antineutrino-to-neutrino ratio, $R^o = \sigma_o^{\bar{\nu}N}/\sigma_o^{\nu N}$, for the neutral-current cross sections is noticeably different from the value of 0.377 measured for the charged-current ratio R^{ch} so long as $\sin^2\theta_W \gtrsim 0.20$. This fact plus the relatively large values for $\sigma_o^{\bar{\nu}N}/\sigma_{ch}^{\bar{\nu}N}$ should play a key role in helping the experimentalists to discriminate a true neutral-current reaction from background if the analysis presented here has any physical significance at all.

It is a pleasure to acknowledge here the valuable mini-conference on partons held at the National Accelerator Laboratory in December 1972 which stimulated the author's interest in the quark parton model, especially the discussions of Professor R. P. Feynman.

$\sin^2\theta_W$	$\sigma_o^{\nu N}$	$\sigma_o^{\bar{\nu} N}$	$\sigma_o^{\bar{\nu}N}/\sigma_o^{\nu N}$	$\sigma_o^{\nu N}/\sigma_{ch}^{\nu N}$	$\sigma_o^{\bar{\nu}N}/\sigma_{ch}^{\bar{\nu} N}$
0	0.226	0.076	0.33	0.51	0.47
0.20	0.152	0.062	0.41	0.34	0.38
0.33	0.120	0.070	0.58	0.27	0.43
0.40	0.109	0.080	0.73	0.25	0.49
0.60	0.099	0.130	1.31	0.22	0.80

Table I. Predictions for the neutral-current neutrino and anti-neutrino cross sections in units of $G^2 s/2\pi$. Ratios of these cross sections relative to each other and to the charged-current cross sections are also given.

Note added in proof:

The quark parton model phenomonologically fitted to the charged-current cross section data by Gourdin[12] involves an equipartition of the quark momenta. If we replace our simple model by that of Gourdin, we find that the new predictions for the neutral-current cross sections differ from those in Table I by less than two per cent.[7]

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[8]Weinberg used the 4-quark model [S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D2, 1285 (1970)] in order to eliminate the

appearance of strangeness-changing neutral currents which are unobserved experimentally to very high precision.

[9]It may be that this angle will be determined from deep theoretical reasoning, but for the moment it appears that the Weinberg and Cabibbo angles are on the same footing. Attempts to determine the latter theoretically are only partially satisfactory.

[10]No evidence exists for charmed quarks in the nucleon, and so we prefer to use the simpler triplet constituent quark model rather than the 4-quark model. Gell-Mann has pointed out at some length the difference between current quarks and constituent quarks, cf. his summary talk presented at the XVI International Conference on High Energy Physics, Chicago - Batavia, 1972.

[11]We use the notation of M. Gourdin, lectures presented at the International School of Physics "Ettore Majorana," Erice 8 - 26 July, 1971 and Nucl. Phys. 29B, 601 (1971).

[12]This picture agrees quite well with that extracted phenomenologically from present data by M. Gourdin, Orsay preprint to be published.

[13]See ref. 12.

[14]Only the sum of d_3 and d_{-3} is determined in the model, but we note that the zeroth moments, $\langle N_3 \rangle$ and $\langle N_{-3} \rangle$ must certainly be equal as the proton carries zero strangeness.

[15]See ref. 3 and the report of Ph. Heusse at the XVI International Conference on High Energy Physics, Chicago - Batavia, 1972.

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